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ONTARIO

DEPARTMENT OF EDUCATION

MATHEMATICS

Intermediate Division

Grade 7

For introduction on an optional basis in September, 1964

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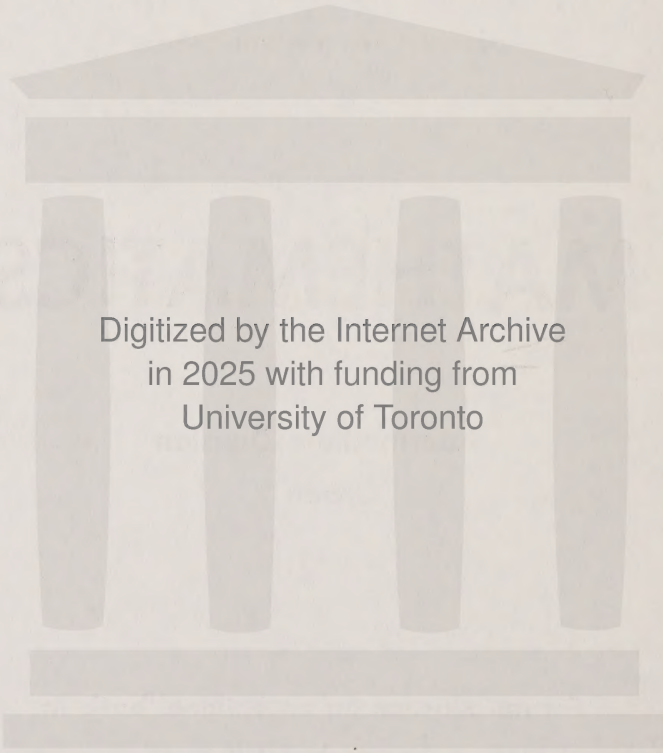
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MATHEMATICS

Grade 7

General Announcement

Beginning in September of 1964, schools will have the option of using this new course or continuing with the course outlined in Curriculum I. 1 (1951) Revised.

General Introduction

This course presents certain changes in content and approach. This introduction attempts to answer some general questions regarding the changes.[1]

A. Why are Changes Needed?
B. In What General Direction are the Changes Needed?
C. What are Some of the General Changes?
D. What is Not Changing?

A. Why Are Changes Needed?

Many questions are being asked about the new approach in mathematics. Are the implied changes either advisable or necessary? Teachers are aware that some new and different approaches are recommended. They are willing to adjust their techniques provided there are good, sound reasons for a change. This introduction attempts to give some reasons for change.

Men are orbiting the earth, satellites are rocketing into space, astronauts are preparing to go to the moon, high-speed electronic computers are whirring — all as a result of the great explosion of knowledge that is taking place in our modern era.

This great explosion of knowledge has affected mathematics in two ways. First, because mathematics itself has changed through new developments within the subject, the additional understanding and appreciation therefrom call for some changes of emphasis at all levels of our school system. Secondly, the contributions of mathematics to discoveries in other branches of knowledge have led to uses and demands for mathematics that were not heard of or even thought of a few years ago. Chemistry and physics are finding new uses and interpretations for mathematics, biology is fast becoming a mathematical subject, business

is using mathematics in scheduling and distribution, sociology is using mathematical statistics, automation is using mathematics for design and for interpretation. Even the citizen who does not use mathematics directly needs to know enough of the subject to appreciate its values and limitations so that he may be an intelligent voter, an understanding reader of current news and literature, a wise manager of a household, and an active participant in the moulding of our civilization in an age of mathematized culture.

The fact that children are staying longer in our schools forces the building of a more basic foundation for mathematics, and at the same time allows postponing of some topics to later grades where added maturity will make possible more effective presentation. That children are staying in school longer is supported by recently published statistics showing that the ratio of population of 15-19 years in Ontario in secondary schools has increased to 71.6% in 1962 from 42.3% in 1951 (the year of publication of Curriculum I. 1.).[2]

B. In What General Direction Are The Changes Needed?

Because of the importance of other subjects in our modern curriculum, mathematics probably cannot claim a larger share of the total curriculum time than it enjoys at present. The only alternative is to teach it more efficiently. What is taught at each stage must contribute to and not conflict with what follows. The selection of topics must be made in the light of modern needs and take account of the fact that pupils will be in school longer. Relationships among various topics must be exploited so that knowledge of one contributes to knowledge of the others.

Furthermore, the very rapidity of change means that it is not clear just what the specific applications and methods will be a few years hence. Thus it is important that our students at all stages of their development be in possession of the fundamental facts, ideas, and methods of mathematics so that they will be prepared to learn, when the time comes, the specific methods required by the job at hand. Above all, they must have the fundamental knowledge and point of view which will enable them to learn any additional mathematics which they will need in their

later years. A man's place in society will be determined not only by what he knows, but by his ability to learn what is needed for his time.

C. What Are Some Of The General Changes?

1. Some topics and points of view are appearing earlier. Formal algebra is out of place in elementary school, but arithmetic taught from an algebraic viewpoint not only deepens understanding of the properties of numbers but prepares for the algebra which is to come. Formal deductive geometry will remain as a part of higher grade work but as soon as the child becomes aware of the shapes of things he can appreciate the meaning of certain kinds of geometrical figures. It is becoming increasingly apparent that ideas like these are not only within the grasp of elementary school students but are of great interest to them.
2. A number of topics which seem new are merely being restored. Examples of such are factoring, least common multiple, and highest common factor. These are being reintroduced, although with a slightly different stress, first, because they involve fundamental properties of the number systems, and secondly, because the student who takes algebra will meet these same topics and will be better able to appreciate and to capitalize on the common elements of arithmetic and algebra.
3. Some topics are new. Numeration systems to different bases may have an appeal because they form the basis for the construction of many of the computing machines. However, a more important reason is that they help the student to consider more carefully just what he is doing when he works with numbers expressed in the decimal system. Inequalities is another topic which has been given little attention in mathematics, even though in real life, the *greater-thans* and the *less-thans* are vitally important.
4. Applications find their place as examples of mathematical principles, rather than as separated topics. For example, instead of teaching the topics of profit and loss, discount, commission, and annual interest as separated topics, the calculation of the amounts

involved in each of these will come in as a part of the exercise immediately following the development of the mathematical principle involved in finding the per cent of a number. This procedure is an outgrowth of the realization that, in rapidly changing times, applications which are important today may be unimportant a few years hence. The object, is, therefore, to educate the student so that he understands the basic principles and can make whatever applications are appropriate to his time.

5. There is some change in language. Some new terms are introduced to increase precision, some to avoid ambiguity, some to simplify expression, and some to give greater depth of understanding. At the same time, where none of these purposes is served, some terms are being eliminated.

6. Stress is laid upon a somewhat new method of approach. Perhaps the most important characteristic of the new approach is the emphasis given to *discovery*, by which pupils are encouraged to explore the relationships between numbers, and between operations. The discovery approach aims at greater competence with number through a higher level of understanding based upon the appreciation of these relationships.

7. A different type of *rule* is finding a place in mathematics and is changing the subject. Traditionally, rules in mathematics have been mainly those which might be called *procedure rules* because they indicated the proper routine to follow in order to arrive at a correct answer. Actually, there is hope that the greater appreciation of number and its relationships will make these rules fewer, and less important for arriving at a correct answer. In place of these *procedure rules* prominence is being given to a different type of rule arising in the presentation of number and the operations of addition, subtraction, multiplication, and division as a game, subject to certain rules which govern what may or may not be done under certain circumstances. Popularly, these *governing* (as distinguished from procedure) *rules* are often referred to as *ground rules*, in order to capitalize upon mathematics as a game with ground rules — which, after all, it really is!

D. What Is Not Changing?

The objectives are not changing! In 1951 the objectives were set down on pages 144 and 145 of Curriculum I. 1, and although their order of importance and their interpretation have undergone some changes, their current applicability cannot be questioned. The 1951 objectives are, therefore, set down here.

1. A thorough understanding of the fundamental operations and increasing power to apply them with accuracy and facility
2. A continuous development of the capacity for mental calculation
3. Ability to apply mathematical knowledge to the solution of problems which are meaningful to the pupil and of social value
4. Self-reliance which comes from willingness to attack and ability to solve problems
5. Acquisition of the habit of judging the reasonableness of an answer and of checking its accuracy
6. The drawing of valid conclusions from experiments in simple space relationships
7. An understanding of the generalization of number and of the application of algebra to simple problems
8. An appreciation of the wide application of mathematics and of its influence in the advancement of civilization
9. A sense of personal responsibility for accuracy, neatness, and precision, with the consequent feeling of satisfaction resulting from work well done
10. The discovery and development of individual mathematical abilities, aptitudes, and interests so that pupils will desire to continue the study of the subject

[1] Among the publications that contain stimulating discussions of these issues are *Why the Need for Modern Arithmetic,* Number 8 bulletin of *The Resourceful Teacher,* published by Silver Burdett, and *The New Mathematics Curricula — What and Why,* by Schneider and Jones, published by the National Council of Teachers of Mathematics. Some of the material in this introduction has been quoted or adapted from these two publications and a general acknowledgement is made herewith.

[2] Table IX from *The Problem of Numbers in University Enrolment*, by R. W. B. Jackson (Bulletin Number 18, 1963), published by the Department of Educational Research, Ontario College of Education, University of Toronto

GRADE 7 COURSE OUTLINE

General Observations On The Arrangement

This outline for Grade 7 is organized in three sections, namely:

Section A: The Study of Whole Numbers
Section B: The Study of Fractional Numbers
Section C: The Study of Geometry and Measurement

Section A is divided into four units, Section B is divided into three units, and Section C is divided into two units.

It must be understood that, for the sections or the units (or indeed for the detail within the units), the particular order indicated is only one of the possible and acceptable orders.

SECTION A: THE STUDY OF THE SET OF WHOLE NUMBERS

During the first six grades of elementary school the pupil has had some considerable experience with the arithmetic of whole numbers. The aim of this section is to examine the properties of the whole numbers and their operations as a mathematical number system. This examination has the main purpose of establishing a model of a mathematical system which can be used as the basis for the study of other types of number, but it also extends into applications made possible by this closer look at these whole numbers.

Unit 1: A Study Of The Properties Of Whole Numbers (The Natural Numbers and Zero)

(Suggested time for this unit: 7 to 8 weeks)

The purpose of this unit is to specify the whole numbers as a set, to use the number-line method of picturing one or

more whole numbers, and to establish a mathematical language for the various properties of whole numbers used in the basic operations of addition, subtraction, multiplication, and division.

1. Development, with examples, of the concept of various numerals for the same number
2. (a) Simple examples of sets of places, people, and numerals to establish the concept of set and the standard method of writing sets using braces

   Use of three dots to indicate *and so on*

   (b) Establishment of the term *natural numbers* as being synonymous with *counting numbers*

   Writing these as a set

   (c) Establishment of the term *whole numbers* as being synonymous with the *natural numbers and zero*

   Writing these as a set

   (d) Writing various subsets of the set of whole numbers,

   e.g., those less than 8, greater than 5, equal to or greater than 4, equal to or less than 9, not equal to 4

   Use of the symbols for: greater than, less than, equal to and greater than, equal to or less than, not equal to

   (e) Picturing of various subsets (as from d) on a number line

   (f) Representation of addition, subtraction, multiplication, and division, by use of segments on a number line
3. Inductive development of the commutative property of addition and multiplication

   Recall of examples from previous work of addition and multiplication where this property was used but not named at the time
4. A treatment of the associative property similar to that of the commutative property

   A similar treatment of the distributive property
5. Development of the fact that subtraction is the inverse operation of addition, and that division is the inverse operation of multiplication

6. Direction of attention to the fact that, in either addition and multiplication of any two whole numbers, the answer is *always* a whole number, but that this is not true for subtraction or division
7. (a) Properties of zero in addition, multiplication, subtraction, and division
   (b) Properties of unity for multiplication and division

Unit 2: The Use Of Number Sentences

(Suggested time for this unit: 3 to 4 weeks)

The purposes of this unit are to establish familiarity with number sentences, to practise finding their solutions by inspection, and to study their application in problem solving.

Notes:

(1) Teachers will be most familiar with the type of *number sentence* which is the statement of an equality of two numerals (familiarly known as an equation). However, relations involving inequality signs (greater-than or less-than) may also be called number sentences. *Number sentences* may also include statements of equality or of inequality, either of which may be true or false. Although most of the problems encountered at the Grade 7 level will be those leading to the type known as the equation, some practice is suggested for the other types of statement included in the term *number sentence.*

(2) The details of this unit include the term *number phrase* (that is, part of a number sentence without the verb). It is to be understood that *number expression* may be used as synonymous with *number phrase.*

(3) Problems should be used in introducing and applying the ideas in this unit.

(4) In this unit it must be remembered that all evaluations will involve whole numbers only, and that, in solutions, whole numbers only will be considered.

Detail for this Unit

1. (a) Introduction with real problems of number phrases indicating more than one operation (e.g.,

Express as a number phrase the amount of money a boy now has who had ten cents and then sold five marbles for three cents each)

(b) Discussion of order of simplifying number phrases (e.g., Use with the above example the number phrase arising from the problem — a boy with ten marbles is given five more and he sells the whole set at three cents each)

(c) Extension to number phrases involving parentheses

(d) Practice in simplifying number phrases involving various operations

2. The use of place holders (screens or letters) for numerals in number phrases

   Establishment of the term *open number phrase*

   Practice in evaluating open number phrases, given specified values for the place holders

3. The form of open number sentences involving equalities

   The form of open number sentences involving inequalities

   Considerable practice at the solving of these by inspection

   A few examples requiring the recording of the solution sets on a number line

Unit 3: A Study Of Factoring

(Suggested time for this unit: 2 to 3 weeks)

The purpose of this unit is to establish the concept of *factor* and *multiple*, and to teach applications of these.

1. (a) Expressing whole numbers as the product of two or more factors

   (b) Classifying of whole numbers in the set of 2 to 100 as prime or composite (Note that, for various reasons, it is best not to classify 1 as either prime or composite)

   (c) Expressing whole numbers as the product of two or more *prime* factors

   (d) Use of the sets of the prime factors of two or more numbers to establish the concept of *common factor* and then of *highest common factor*

2. (a) Practice in writing the sets of multiples of simple whole numbers

   (b) Writing the *lowest common multiple* by inspection of the sets of multiples of two or more whole numbers

.3. (a) Special study of whole numbers which give a repetition of the same factor, e.g., 2 x 2 x 2, 3 x 3 x 3 x 3, 10 x 10 x 10 x 10 x 10

   (b) Use of an index (or exponent) to indicate the number of times the same factor appears

Note: Encourage the reading of these short forms as "2 to the index 3", or "2 to the exponent 3", rather than allowing the use of *power*. *Power* includes both the base (the repeated factor), and the index, and is not synonymous with *index*

   (c) Practice in evaluating given powers

   (d) Practice in writing special whole numbers as powers, e.g., 4, 9, 16, 27, 32, 36, etcetera

   (e) Evaluation of number phrases involving powers.

4. (a) The review of place value in whole numbers in the decimal numeration system

   (b) Writing of whole numbers in expanded notation, that is, 2346 as $2 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 6 \times 1$

Unit 4: Numeration Systems To Bases Other Than Ten

(Suggested time for this unit: 2 to 3 weeks)

The main purpose of this unit is to clarify the understanding of the decimal system of numeration. Experience with this topic has indicated that it is also of interest to pupils because it contributes to understanding of the basic principle of the computers.

Note: It should be noted that, for this unit particularly, there are several ways of approach. Some ways of approach will allow this unit to be placed almost anywhere in the year's work. In fact, if time difficulty is encountered in the first year of teaching this course, this topic can be delayed without any serious sequence problems.

1. A review of place value in whole numbers in the decimal numeration system

   A study of grouping of objects (and dots) to show groups of ten and how groups of ten tens are recorded as sets of hundreds

   A recall of the writing of these whole numbers in expanded notation as $2 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 6 \times 1$
2. Introduction of groupings similar to above using some base other than ten

   Expression of the total of these groups in expanded notation

   Conversion from this expanded notation to a numeration using this other base
3. Practice in converting from numerals in the base ten system to numerals in this other base

   Practice in converting from numerals in this other base to numerals in the base ten system
4. Some practice in addition and subtraction of numerals in this other base
5. (Supplementary) Repetition of details of 2, 3, 4 above with still other bases

SECTION B: THE STUDY OF THE SET OF FRACTIONAL NUMBERS

The aim of this section is to study the fractional numbers. In the first unit, a careful new look is taken at the meaning of fractional numbers, and the operations with them as learned in previous grades. Then the work is extended to give meaning to, and facility with, further operations. In the second unit, the decimal notation is introduced as another way of writing the names of fractional numbers, and the operations in this notation are studied. In the third unit, per cent notation is studied as still another way of naming fractional numbers. Further, a study is made of those properties that al'ow the classification of the set of fractional numbers as a mathematical number system.

Note: 1. *Fractional numbers* are numbers named by the use of a pair of whole numbers, for example $\frac{3}{4}$

2. *Fractions* are the various numerals for the fractional number and they appear in three notations,
   (i) the fraction notation, for example $\frac{3}{4}, \frac{9}{12}, \frac{12}{16}$
   (ii) the decimal notation, for example .75
   (iii) the per cent notation, for example 75%
3. Traditionally, the names, *fractions*, *decimals*, and *per cents* have been used to stand for both the number and the numeral, depending on the context to make it clear what is meant. To avoid over-emphasis on rigour at this stage, the traditional practice may be continued.

Unit 1: The Common Fractions

(Suggested time for this unit: 5 to 6 weeks)

1. A review of the form of fractions
   Extension to the general form $\frac{a}{b}$
   Reference to subsection 7 (a) of Unit 1, Section A, to indicate the need for excluding the case where b is equal to zero
2. A discussion of the form $\frac{a}{b}$ to show that this has been used, not only to indicate the parts of a whole or of a group, but also for comparisons and for rates
3. Multiplication of fractions
   (a) Product of a whole number and a fraction
   Product of two fractions
   (b) Study of multiplication to indicate the commutative, associative, and distributive properties
   (c) Review of subsection 6 of Section A, Unit 1, to indicate that the set of common fractional numbers is a set in which division is always possible
4. (a) Review of the Principle of Equivalent Fractions
   (b) Application of this principle to
      (i) changing fractions to higher terms
      (ii) changing fractions to lower terms

5. Division of common fractions
6. Review of the addition of common fractions
(Treatment to include horizontal arrangement)
7. Study of the commutative, associative, and distributive properties, with respect to addition, and multiplication, to discover that they apply in all the operations studied
8. Review of the subtraction of fractions with stress on subtraction as the inverse operation to addition
9. Comparison of fractions
Finding fractions between fractions
Graphing some of these fractions between fractions on a number line
10. Problems involving all operations with fractions
11. (Supplementary)
   (a) Simple cases of solution by inspection of equations where the solution may be a fraction
   (b) A few very simple cases of solution by inspection of inequalities where the solutions may include fractions

Unit 2: The Decimal Notation

(Suggested time for this unit: 5 to 6 weeks)

1. (a) Review of expanded notation of whole numbers, e.g., $222 = 2 \times 10^2 + 2 \times 10 + 2 \times 1$
   (b) Extension of pattern to such as,
   $2 \times 10^2 + 2 \times 10 + 2 \times 1 + 2 \times \frac{1}{10} + 2 \times \frac{1}{10^2} + 2 \times \frac{1}{10^3}$
   and then this recorded in place value as 222.222
   (c) Practice in writing decimal numerals in expanded notation and vice versa
2. Addition and subtraction of decimals
(Arrangements *in line* treated as means to get various values in line, rather than just the "decimal points")

3. (a) Multiplication of decimals with proper place value determined by relating to multiplication in common fraction form
   Later determination of same by estimation

   (b) Considerable practice in multiplying decimals where one factor is a power of ten

   (c) (Supplementary)
   Writing of decimals as factors where one factor is a power of ten and the other factor is a number between 1 and 10, e.g., $623.4 = 6.234 \times 10^2$

4. Division of decimals (stressing the use of the Principle of Equivalent Fractions)

5. (a) Changing of common fractions to decimals

      (i) where the denominator is a power of 10

      (ii) where the denominator involves the prime factors 2 and 5 only

      (iii) other fractions

   (b) Rounding off decimals to a specified number of decimal places

6. Use of decimal notation on a number line
   Placing decimals on a number line as a special case of fractions between fractions
   Discussion of the application of this procedure in scaling of measuring devices, such as rulers, etc.

7. Problems involving various operations with decimals

Unit 3: Use Of The Fractional Number For Comparisons and Extension To Per Cent

(Suggested time for this unit: 3 to 4 weeks)

1. Review use of the fraction form to show parts of whole, and for comparisons

2. (a) Use of the fraction form to indicate ratio

   (b) Determination of equivalent ratios

   (c) Drawing of parallel between (b) and the Principle of Equivalent Fractions

(d) Problems involving equivalent ratios

(e) Determination of the test for equivalent ratios
Further problems applying this test

3. (a) Introduction of *per cent* as a special two-term ratio with the second term being 100

(b) Changing of *per cent* ratios to equivalent ratios, and vice versa

(c) Use of equivalent ratios in various problems involving numbers

(d) Use of equivalent ratios in various problems involving gain or loss, discount, simple annual interest, and commission

4. Introduction to picto-graphs and bar-graphs to illustrate comparisons (stress the interpretation of the graphs, rather than their construction)

SECTION C: GEOMETRY AND MEASUREMENT

(Suggested time for this section: 5 to 6 weeks)

This section is to introduce some of the simpler language relating to straight lines and straight line figures, and to extend the experience from previous grades of measurements of these figures.

Unit 1: Some Descriptive Geometry Of Straight Line Figures

1. A discussion of the geometrical concept of a *point*
2. An intuitive approach (using the idea of sets of points) to the following geometrical concepts: *(straight) line, ray, line segment, plane*

   Discussion of the intersection of two lines in the same plane

   The idea of parallel lines as a case of the non-intersection of two lines *in a plane*
3. The idea of an *angle* as the set of points on two rays which have a common end point

Note: It is important that when we speak of an angle as related to the *amount of rotation*, we are using the *measure* idea of an angle. Often, however, we refer to angle when the two rays from the point constitute the only significant feature.

4. (a) A study of simple closed figures

   (b) A classification of simple closed figures according to number of sides — *triangle, quadrilateral, pentagon, hexagon*

Unit 2: Measurement Related To Straight Line Figures

1. (a) The measure of a *line segment*

   (b) The study of the set of triangles according to relative measure of sides (equilateral, isosceles, scalene)

   (c) The study of the set of quadrilaterals according to relative measure of sides (square, rectangle, parallelogram)

2. Review of the concept of perimeter

   Application of perimeter to triangles, quadrilaterals, and other polygons (no formula or rule).

   Many problems based thereon

3. (a) The measure of an angle

   (b) The classification of the set of angles according to measure (straight, right, acute, obtuse)

   (c) The classification of the set of triangles according to measure of the largest angle (acute-angled, obtuse-angled, right-angled)

4. Ruler-compasses construction of line-segments, angles, triangles

5. Review of the measure of area of the rectangle and the square, and extension to the parallelogram and triangle

   Problems based thereon

6. The measure of the volume of a rectangular solid, and the units involved in the English system

   Problems based thereon